Mathematics: analysis and approaches
Higher level
Paper 1

Name

Date: ____________________

2 hours

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

exam: 12 pages

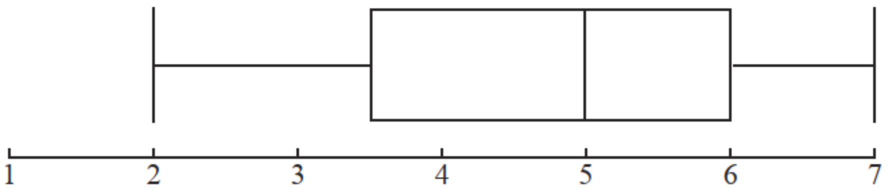
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A (56 marks)

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The box and whisker diagram below illustrates the IB grades for a group of 20 students.
IB grades are an integer from 1 to 7. The mode grade is 6.



(a) Write down the median grade. [1]

(b) Find the number of students who obtained a grade greater than 3. [2]

(c) Determine, with a reason, the maximum number of students who could obtain a grade of 7. [2]

2. [Maximum mark: 6]

The angle θ lies in the first quadrant and $\sin\theta = \dfrac{1}{3}$.

(a) Write down the value of $\cos\theta$. [1]

(b) Find the value of $\cos 2\theta$. [2]

(c) Find the value of $\tan 2\theta$, giving your answer in the form $\dfrac{a\sqrt{b}}{c}$ where $a, b, c \in \mathbb{Z}^{+}$. [3]

3. [Maximum mark: 6]

If $y = x^2 \ln(x)$,

(a) find the x-coordinate of the point M where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$; [3]

(b) determine whether M is a maximum or minimum point. [3]

4. [Maximum mark: 7]

A game consists of a contestant rolling three fair six-sided dice. If a 4, 5 or 6 turns up on any of the three dice, then the contestant loses $\$2$. If none of the dice turn up a 4, 5 or 6, then the contestant wins $\$20$.

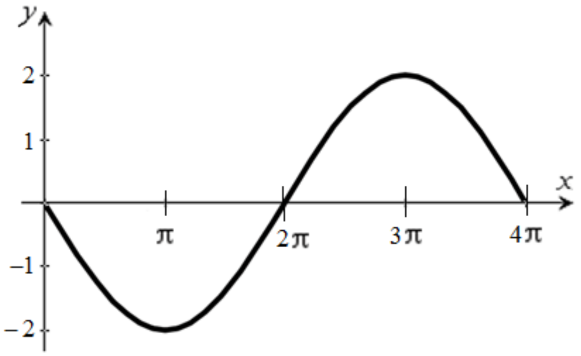
(a) Show that the contestant expects to win \$3 if the contestant plays the game four times. [4]

One change is made to the game. If none of the dice turn up a 4, 5 or 6, then the contestant wins x dollars.

(b) Find the value of x so that the game is fair. [3]

5. [Maximum mark: 7]

The graph of $f(x) = a\cos[b(x-\pi)]$ for the interval $0 \le x \le 4\pi$ is shown below.



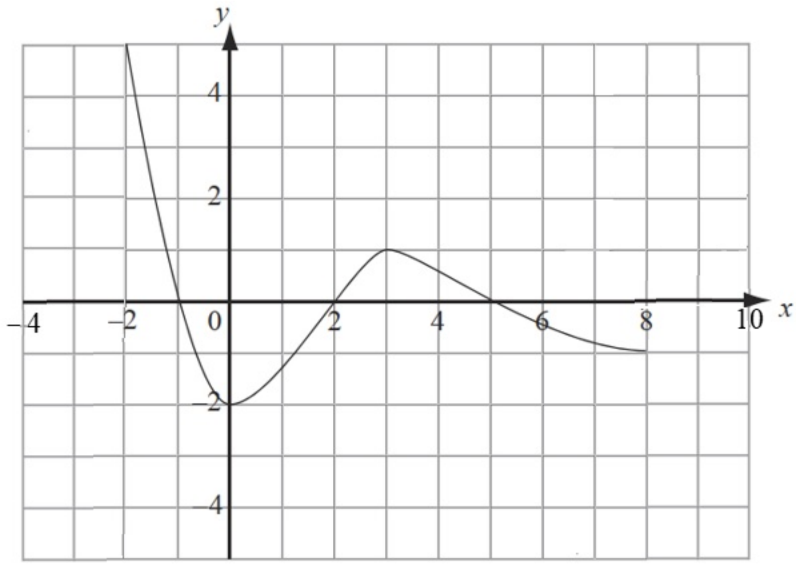
(a) Write down the value of a and the value of b. [2]

(b) Find the gradient of the graph of f at $x = \frac{3\pi}{2}$. [3]

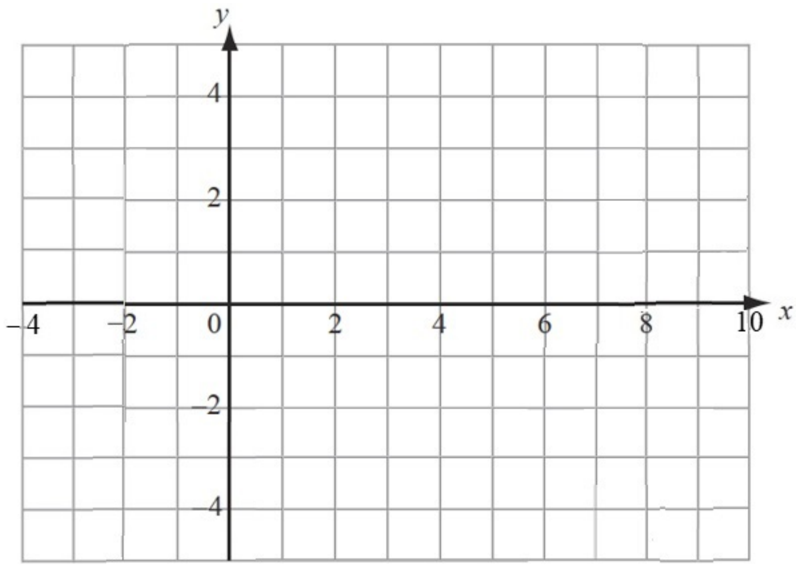
(c) Given that $0 \le c \le 4\pi$, explain why $\int_c^{4\pi - c} f(x)\,dx = 0$. [2]

6. [Maximum mark: 7]

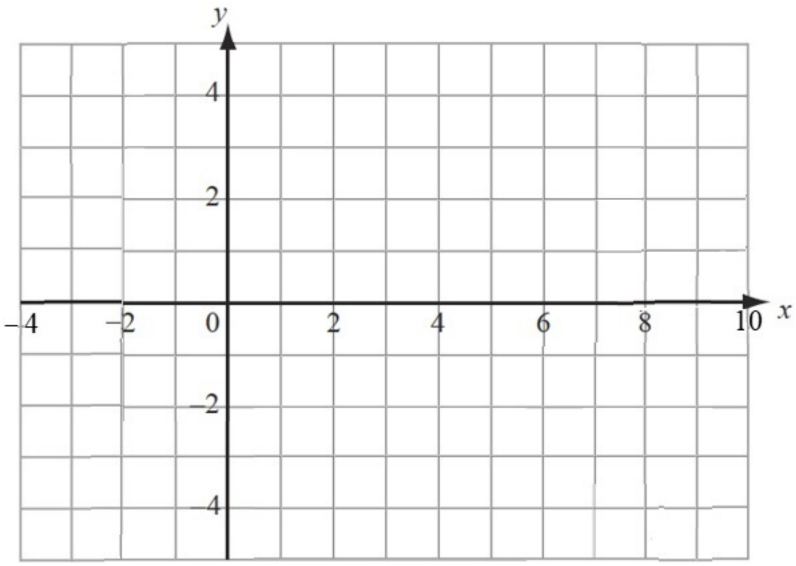
The graph of $y = g(x)$ is shown.



(a) On the set of axes below, sketch the graph of $y = \dfrac{1}{g(x)}$, clearly showing any asymptotes and indicating the coordinates of any local maxima or minima. [4]



(b) On the set of axes below, sketch the graph of $y = g(2x-2)$, clearly showing any asymptotes and indicating the coordinates of any local maxima or minima. [3]



7. [Maximum mark: 6]

Prove, using mathematical induction, that for any positive integer n,

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$ [6]

8. [Maximum mark: 7]

Solve the following differential equation. Write your solution as an equation where y is expressed in terms of x.

$$\frac{dy}{dx} + 3x^2 y = \left(1 + 3x^2\right)e^x$$ [7]

9. [Maximum mark: 5]

Given that $k > 0$, find the values of k such that $kx^2 - 4x + k + 3 > 0$ for all real values of x. [5]

Do **not** write solutions on this page.

Section B (54 marks)

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

10. [Maximum mark: 16]

In a class of 85, all of the students must study French or Spanish. Some of the students study both French and Spanish. 51 students study French and 43 students study Spanish.

(a) (i) Find the number of students who study **both** French and Spanish.

(ii) Write down the number of students who study **only** Spanish.

(iii) Write down the number of students who study **only** French. [4]

One student is selected at random from the class.

(b) Find the probability that the student studies **only** one language.

(c) Given that the student selected studies **only** one language, find the probability that

(i) the student studies Spanish;

(ii) the student studies French. [6]

Let F be the event that a student studies French and S be the event that a student studies Spanish.

(d) Determine, with explanation, whether

(i) F and S are **mutually exclusive** events;

(ii) F and S are **independent** events. [6]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

Consider the complex numbers $z_1 = 2\text{cis}\frac{5\pi}{6}$ and $z_2 = -1 + \text{i}$

(a) Calculate $\frac{z_1}{z_2}$. Express your answer in both modulus-argument form and Cartesian form. [8]

(b) Prove that $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$. [3]

(c) Using your results from (a) and (b), find the exact value of $\tan\frac{5\pi}{12}$. Express your answer In the form $a + \sqrt{b}$, where $a, b \in \mathbb{Z}^+$. [6]

12. [Maximum mark: 21]

(a) Obtain the Maclaurin series for $f(x) = \text{e}^{2x}$ up to, and including, the x^3 term. [5]

(b) Let $g(x) = \tan x$.

(i) Find an expression for $g'(x)$, $g''(x)$ and $g'''(x)$.

(ii) Hence, obtain the Maclaurin series for $g(x)$ up to, and including, the x^3 term. [9]

(c) Hence, or otherwise, obtain the Maclaurin series for $\text{e}^{2x}\tan x$ up to, and including, the x^3 term. [2]

(d) Find the first four non-zero terms in the Maclaurin series for $2\text{e}^{2x}\tan x + \text{e}^{2x}\sec^2 x$. [5]